A Hierarchical Bayesian Model of Consumer Learning in Online Penny Auctions

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Outline

- Introduction
- Research objective
- Empirical model
- Data
- Estimation
- Results
- Future research
Introduction

A PA example from dealdash.com

- Retail Price: ¥4,899
- The auctioneer’s revenue: ¥13,872+ ¥138.72
- The winner made 670 bids in this auction
- The winner paid ¥3*670+ ¥138.72
- Losers paid bidding fee but got nothing
  - One loser made 875 bids, the bidding fee is ¥3*875
Introduction

How is the mechanism different?

- **Traditional Auctions:**
  - if you win, you get the product with price close to its market value.
  - if you lose, it does not cost you anything.

- **Penny Auctions:**
  - if you win, you could achieve huge savings.
  - If you lose, you get nothing after spending a lot in bids.
Research Objective

Prior research and our objective

- Our research objective:
  - Using **learning** to help understand bidders’ behavior better.

- Ma et al. (2013). Structural model of consumer behaviors.

- Wang and Xu (2012). **Learning.** PA is **not sustainable.**

- Platt et al. (2011). **Predicts** distribution of ending prices.

- Augenblick (2011). Bidders’ behaviors. Do they **learn**?

- Byers et al. (2010). Information Asymmetries, **Auctioneers win.**

- Hinnosaar (2010). Penny Auctions are **Unpredictable.**
Data

Data set

Data collection:
Using http-watcher to monitor the webpage and doing data cleaning

- Item Name
- Retail Price
- Closing Price
- Commission Fee
- Bid Cost
- …

- Auction Level
  - Auction ID
  - Starting Price
  - Ending Price
  - Price Increment
  - …

- Bid Level
  - Bid ID
  - Time Placed
  - Active Player Total
  - …
### Descriptive statistics

#### Revenue distribution

**Table 2: Auctioneer Revenue from Auctions by Product Categories**

<table>
<thead>
<tr>
<th>Product Categories</th>
<th>No. of Auctions</th>
<th>Total Revenue</th>
<th>Total Value of Products Sold</th>
<th>Revenue Per ¥ Worth of Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bidding Tokens</td>
<td>3,344</td>
<td>867,561</td>
<td>138,090</td>
<td>6.28</td>
</tr>
<tr>
<td>Apple Products</td>
<td>304</td>
<td>2,081,011</td>
<td>646,484</td>
<td>3.22</td>
</tr>
<tr>
<td>20,30,50,100,200 Refill Cards</td>
<td>2,438</td>
<td>403,266</td>
<td>146,220</td>
<td>2.76</td>
</tr>
<tr>
<td>Notebooks</td>
<td>82</td>
<td>455,562</td>
<td>205,041</td>
<td>2.22</td>
</tr>
<tr>
<td>Mobile Phones</td>
<td>403</td>
<td>808,611</td>
<td>383,920</td>
<td>2.11</td>
</tr>
<tr>
<td>PCs and Monitors</td>
<td>126</td>
<td>378,507</td>
<td>205,154</td>
<td>1.84</td>
</tr>
<tr>
<td>GPS Navigators</td>
<td>164</td>
<td>81,201</td>
<td>96,957</td>
<td>0.84</td>
</tr>
<tr>
<td>Portable Media Players</td>
<td>749</td>
<td>207,712</td>
<td>248,088</td>
<td>0.84</td>
</tr>
<tr>
<td>Audio Players</td>
<td>321</td>
<td>82,616</td>
<td>109,225</td>
<td>0.76</td>
</tr>
<tr>
<td>Coupons</td>
<td>873</td>
<td>24,230</td>
<td>36,986</td>
<td>0.66</td>
</tr>
<tr>
<td>10 Refill Cards</td>
<td>2,771</td>
<td>14,194</td>
<td>27,710</td>
<td>0.51</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>23,884</strong></td>
<td><strong>8,576,823</strong></td>
<td><strong>5,111,839</strong></td>
<td><strong>1.68</strong></td>
</tr>
</tbody>
</table>
Descriptive statistics

Auction types

<table>
<thead>
<tr>
<th>Category</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Virtual Products</td>
<td>General Merchandise</td>
<td>Cheap Digital Products</td>
<td>Expensive Digital Products</td>
<td>Free Auctions</td>
<td>Total</td>
</tr>
<tr>
<td># Auctioned</td>
<td>5,486</td>
<td>5,142</td>
<td>6,790</td>
<td>479</td>
<td>3,566</td>
<td>21,463</td>
</tr>
<tr>
<td># Profitable</td>
<td>3,975</td>
<td>1,600</td>
<td>2,095</td>
<td>317</td>
<td>7</td>
<td>7,994</td>
</tr>
<tr>
<td>% Profitable</td>
<td>0.725</td>
<td>0.311</td>
<td>0.309</td>
<td>0.662</td>
<td>0.002</td>
<td>0.372</td>
</tr>
<tr>
<td>Ave. Retail Price</td>
<td>50.45</td>
<td>220.84</td>
<td>280.31</td>
<td>2,749.10</td>
<td>31.52</td>
<td>221.07</td>
</tr>
</tbody>
</table>
Descriptive statistics

Is the sub-sample meaningful?

Time Period:

Sampling by Participation:
>=50  635 bidders
>=100 194 bidders
Empirical model

Utility and likelihood

- Refer to Erdem and Keane (1996), Erdem et al. (2008) and Ghose and Han (2012), we define the utility as:

\[ U_{ijt} = w_E E_{ijt} - w_E r E_{ijt}^2 + w_M (R_{ijt} - S_{ijt}) + e_{ijt} \]

- Two parts: entertainment part and monetary part;
- Non-linear in entertainment to capture bidders’ risk preferences towards the entertainment they obtain.

- Likelihood: \[ L(A_{ijt}) = \left( \frac{\exp(\bar{U}_{ijt})}{1 + \exp(\bar{U}_{ijt})} \right) A_{ijt} \left( \frac{1}{1 + \exp(\bar{U}_{ijt})} \right)^{1 - A_{ijt}} \]

\( i \) : index the bidder \hspace{1cm} \( j \) : denotes the auction category \hspace{1cm} \( t \) : represent the time unit “day”

\( E_{ijt} \): entertainment value \hspace{1cm} \( R_{ijt} \): monetary benefit \hspace{1cm} \( S_{ijt} \): sunk cost \hspace{1cm} \( e_{ijt} \): capture a bidder’s preference shock

\( w_E \): weight for the entertainment value \hspace{1cm} \( w_M \): weight for the monetary value \hspace{1cm} \( r \): risk coefficient

\( A_{ijt} \): to indicate participating (1) or not (0)
Empirical model

Consumer learning

When bidder registered for the website, their prior belief of the entertainment follows a normal distribution:

$$E_{ij0} \sim N(E_{i0}, \sigma^2_{E_{i0}})$$

$E_{ij}$ is the mean entertainment value bidder $i$ obtains from category $j$ auctions; $\epsilon_{ijtm}$ is the deviation; $\sigma^2_{E_i}$ is individual specific. The $m^{th}$ entertainment signal for consumer $i$ is given by:

$$E_{ijtm} = E_{ij} + \epsilon_{ijtm} \quad \epsilon_{ijtm} \sim N(0, \sigma^2_{E_i})$$

If there are $n_{e_{ijt}}$ category $j$ auctions participated by the bidder on day $t$, then the aggregate signal he/she receives is given as follows:

$$E_{ijts} = \frac{\sum_m E_{ijtm}}{n_{e_{ijt}}} \sim N(E_{ij}, \frac{\sigma^2_{E_i}}{n_{e_{ijt}}})$$
Empirical model

Bayesian updating

According to the normal learning Bayes rule (DeGroot 1970), the posterior belief at every time period follows a normal distribution:

\[ E_{ijt} \sim N(E_{ijt}^e, \sigma_{E_{ijt}}^2) \]

where

\[ E_{ijt}^e = \frac{\sigma_{E_{ijt}}^2}{\sigma_{E_{ij(t-1)}}^2} E_{ij(t-1)}^e + ne_{ijt} \frac{\sigma_{E_{ijt}}^2}{\sigma_{E_i}^2} E_{ijts} \]

\[ \sigma_{E_{ijt}}^2 = \frac{1}{1/\sigma_{E_{ij(t-1)}}^2 + ne_{ijt}/\sigma_{E_i}^2} \]

The prior in period \( t = 0 \) is \( E_{ij0}^e = E_{i0} \), \( \sigma_{E_{ij0}}^2 = \sigma_{E_{i0}}^2 \)
Estimation

Parameters and methods

- Parameters:
  \[ \gamma_i = \begin{bmatrix} E_{i1} & E_{i2} & E_{i3} & E_{i4} & E_{i5} & \ln\sigma^2_{E_i} & E_{i0} \end{bmatrix} \]
  \[ \psi = \begin{bmatrix} w_E & r & w_M \end{bmatrix} \]

- Refer to Netzer et al. (2008) and Narayanan and Manchanda (2009)
  We use hierarchical Bayesian estimation methods:

  Step 1 \[ \gamma_i \mid A_i, E_i, \psi, \bar{\gamma}, V_{\gamma} \] Adaptive MH
  Step 2 \[ \bar{\gamma} \mid \{\gamma_i\}, V_{\gamma} \] Gibbs Sampling
  Step 3 \[ V_{\gamma} \mid \{\gamma_i\}, \bar{\gamma} \] Gibbs Sampling
  Step 4 \[ \psi \mid A, E, \{\gamma_i\} \] Adaptive MH
  Step 5 \[ E_{ijt} \mid A_{ijt}, E_{ijt-1}, \psi, \gamma_i \] Adaptive MH
Results

Parameter Estimates

Table 6: Pooled Parameter Estimates

<table>
<thead>
<tr>
<th>Notation</th>
<th>Parameter Estimates</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_E )</td>
<td>0.150</td>
<td>0.568</td>
</tr>
<tr>
<td>( r )</td>
<td>-1.039</td>
<td>0.342</td>
</tr>
<tr>
<td>( w_M )</td>
<td>0.779</td>
<td>0.383</td>
</tr>
</tbody>
</table>

Table 7: Bidder-level Parameter Estimates

<table>
<thead>
<tr>
<th>Notation</th>
<th>Parameter Estimates*</th>
<th>Standard Deviation*</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{i1} )</td>
<td>0.859</td>
<td>0.122</td>
</tr>
<tr>
<td>( E_{i2} )</td>
<td>1.165</td>
<td>0.136</td>
</tr>
<tr>
<td>( E_{i3} )</td>
<td>0.884</td>
<td>0.147</td>
</tr>
<tr>
<td>( E_{i4} )</td>
<td>1.140</td>
<td>0.152</td>
</tr>
<tr>
<td>( E_{i5} )</td>
<td>0.767</td>
<td>0.117</td>
</tr>
<tr>
<td>( \ln\sigma^2_{E_{i1}} )</td>
<td>1.053</td>
<td>0.125</td>
</tr>
<tr>
<td>( E_{i0} )</td>
<td>1.943</td>
<td>1.724</td>
</tr>
</tbody>
</table>

*For each bidder, the posterior distribution of each parameter has a mean and s.d. The mean and standard deviation listed in the table are the mean and s.d. of the bidder-level parameter means.

1. For \( w_E \), positive sign;
2. Risk seeking;
3. Expected more profit, interested more;
4. Overestimate \( E \) value initially;
5. S.D. of \( E_{i0} \) is larger, bidders vary a lot;
6. Category 2 : various auctions;
7. Category 4 : once win, win large;
8. \( \exp(1.053)=2.866 \), signals are not very accurate.
Results

Major findings
- $w_E$ and $w_M$ have expected signs
- Bidders are risk seeking, so they persist in such a gambling.
- Bidders overestimate entertainments before participating.
- Bidders obtain larger entertainment from type 2 & 4 auctions.

Future research
- Robust test
  - Larger samples
- Policy simulation
  - Information disclose
Thank You!